# ARTICLE VI.

## ON THE NUMERICAL RELATIONS OF GRAVITY AND MAGNETISM.

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THE dependence of magnetism upon currents of some kind, has long been generally admitted; but it has usually been assumed that these currents are of a peculiar nature, and that they require for their development the existence of a magnetic fluid or æther, distinct in its properties from any other form of matter.

A current presupposes an impelling force, competent to produce a tendency of particles toward some particular point, and an inert or resisting medium upon or against which the force is exerted. Motion, whenever it occurs, is always in the direction of least resistance,—a direction that can be calculated by mechanical laws, when all the determining data are known.

That all forces are in some manner mutually connected, so that gravity and magnetism may be more or less convertible into each other, like light, heat, and electricity, was early suspected by Professor Faraday;* but, on account of the impossibility of escaping from the influence of terrestrial magnetism, all attempts to demonstrate their convertibility have been hitherto unavailing. I have elsewhere suggested, as a possible test experiment,† "centrifugal force, so applied as alternately to assist and oppose the effects of gravity, as in large fly-wheels revolving with various degrees of rapidity." An arrangement of this kind is presented by the earth in its daily rotation, its centrifugal force being alternately added and opposed to the influence of solar attraction; and, from an examination of various recorded observations, I have deduced the following propositions:

* Phil. Mag. [4] 1, 68. My own belief in the correlation of motion, light, attraction, heat, electricity, and vitality, was first publicly announced in a lecture delivered before the Lyceums of Worcester and Lynn, Massachusetts, in the winter of 1844–5. During the two previous years, Mayer, Colding, Joule, and Grove, had published their first papers; but I am not aware that any report of their views had reached America.

If we regard mobility under its three necessary relations, we may, perhaps, advantageously group the elementary forces under the three heads of—

1. Influx: Gravitation, Affinity, Cohesion.
2. Immanence: Polarity, Electricity, Vitality.
3. Emission: Light, Heat, Actinism.

† Proc. Am. Phil. Soc. v. ix, p. 356.

I. The daily magnetic variations, though subject to great disturbances at different hours, show an average approximation to the differences of the gravitation-tidal currents.

II. Marked indications of an accelerating force are discoverable in the magnetic fluctuations, especially during the hours when the sun is above the horizon.

III. There are lunar-monthly barometric and magnetic tides, which may be explained by differences of weight or momentum,* occasioned by the combined influences of solar and lunar attraction and terrestrial rotation.

IV. The solar-diurnal variations of magnetism between noon and midnight (.00138), are nearly identical in amount with the variations of weight produced by solar attraction at the same hours (.00134).

V. The magnetic variations at intermediate hours between noon and midnight, indicate the influence of an accelerating force like that of gravity, modified by fluctuations of temperature and by atmospheric or æthereal currents.

VI. Some of the magnetic influences appear to be transmitted instantaneously, through the rapid pulsations of the kinetic æther,—others gradually, through the comparatively sluggish vibrations of the air.

VII. The comparative barometric disturbances of the sun and moon, exhibit an approximate mean proportionality between their comparative differential-tidal and magnetic disturbances.

VIII. The theoretical gravitation-variation of magnetism (Prop. IV) is slightly less, while the theoretical barometric-variation (Prop. VII) is slightly greater than the corresponding observed variation. The excess in one case exactly counterbalances the deficiency in the other, the sum of the theoretical being precisely equal to the sum of the observed variations.

IX. The total daily magnetic variations, like the barometric, can be resolved into a variety of special tides, which may be severally explained by well-known constant or variable current-producing and weight-disturbing forces.

X. The phenomena of magnetic storms indicate the existence of controlling laws analogous to those which regulate the normal fluctuations.

I propose briefly to substantiate these several propositions by a reference to the data on which they are based.

## I.

The mere announcement of any new numerical relations between the effects of two great natural forces, like the first discovery of the inverse ratios of gravity and magnetism to the square of the distance, is curious and interesting, even if it should lead to no

* I believe there can be no weight without some degree of momentum. See Proc. A. P. S., vol. ix, p. 357.

ulterior results, and it may be reasonably supposed that such a ratio betokens a natural connection between the forces themselves. This hypothesis is greatly strengthened in the case now under consideration by the experimental illustrations that I have given of the influence of mechanical vibrations upon the magnetic needle (Proc. A. P. S., vol. ix, p. 359), and by the evident tendency of simple known causes to produce spiral eddies in the air and æther analogous to those which are constantly circulating around permanent magnets. (Ibid., pp. 367, sqq).* Having the elements of a mechanical polarity thus given, we might reasonably suppose that any excess of magnetic influence in one portion of the day would be balanced by a deficiency in some other portion, so that the average hourly differences would assume a tidal form, as stated in Prop. I. That such is the case is shown by

TABLE I. DAILY DIFFERENCES OF MAGNETIC FORCE.†

| DIFFERENCES IN HUNDRED-THOUSANDTHS OF TOTAL FORCE. | | | | RATIOS. | | |
|---|---|---|---|---|---|---|
| Astronomical Time. | 1 h. | 2 h. | 3 h. | 1 h. | 2 h. | 3 h. |
| Before 3 h. | 20 | 42 | 55 | .364 | .764 | 1 |
| After 3 h. | 20 | 39 | 55 | .364 | .709 | 1 |
| Before 9 h. | 5 | 15 | 26 | .192 | .577 | 1 |
| After 9 h. | 4 | 4 | 2 | 2.000 | 2.000 | 1 |
| Before 15 h. | 2 | 5 | 7 | .286 | .714 | 1 |
| After 15 h. | 1 | 2 | 4 | .250 | .500 | 1 |
| Before 21 h. | 31 | 54 | 66 | .470 | .818 | 1 |
| After 21 h. | 29 | 51 | 61 | .475 | .836 | 1 |
| Means, | 14 | 26.5 | 34.5 | .406 | .761 | 1 |
| Mean of Ratios, | | | | .563 | .865 | 1 |

* The simplest of the three experiments referred to is performed with an ordinary binnacle compass. Holding the gimbals so as to allow motion in only one direction, and causing the box to swing like a pendulum, the needle will tend towards the line of oscillation, thus showing that simple mechanical vibrations can produce a species of polarity. If the combined action of convection and rotation upon the solar meridional currents, and on the temperature currents between the equator and the poles of greatest cold, be carefully studied, it will be seen that they furnish the elements of two sets of spiral eddies, or quasi-horizontal cyclones, in the air and æther, one flowing in a nearly constant direction along the magnetic meridian, and the other towards the momentarily shifting solar meridian. As the air has a certain degree of specific magnetism, the polarity which these eddies produce, may explain the directive energy of the compass needle.

† All of these tables are compiled from the St. Helena Observations and General Sabine's Discussions.

## II.

TABLE II. SOLAR-DIURNAL VARIATION IN THE MAGNETIC FORCE AT ST. HELENA, IN HUNDRED-THOUSANDTHS OF THE TOTAL FORCE.

| Time. | 0 h. | 1 h. | 2 h. | 3 h. | 4 h. | 5 h. | 6 h. | 7 h. | 8 h. | 9 h. | 10 h. | 11 h. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A. M. | —43 | —41 | —38 | —36 | —35 | —34 | —32 | —20 | + 3 | +34 | +63 | +85 |
| P. M. | +95 | +82 | +60 | +40 | +20 | + 1 | —15 | —26 | —36 | —41 | —45 | —45 |

A simple inspection of the above table will furnish evidence of the truth of Prop. II. The relation of the magnetic to the tidal variations between 6 A. M. and 6 P. M. is more clearly shown by

TABLE III. HOURLY MAGNETIC DIFFERENCES DURING THE DAY HOURS.

| | 1st h. | 2d h. | 3d h. |
|---|---|---|---|
| Before 9 A. M., . . . . . . . | 31 | 23 | 12 |
| After 9 A. M., . . . . . . . | 29 | 22 | 10 |
| Before 3 P. M., . . . . . . . | 20 | 22 | 13 |
| After 3 P. M., . . . . . . . | 20 | 19 | 16 |
| | 100 | 86 | 51 |
| Tidal ratios, . . . . . . . . | 100 | 73 | 27 |
| Ratios of squares of mean magnetic differences, . . . . . . . . | 100 | 74 | 26 |

For further illustrations, see the discussion of Prop. V.

## III.

If we examine the position of the aerial rotation-spheroid, it will be seen that when the moon is approaching syzygy, at a distance of about 30°, in addition to the normal tidal action due to her situation relatively to the sun, she exerts a compressing force which tends to raise the daily average of the barometer, while, at the same distance before her quadrature, this secondary influence serves to increase the ellipticity and depress the barometric daily average. At intermediate points a portion of her attraction is employed in turning the aerial spheroid, thus producing a condensation of the air and elevation of the barometer, or a rarefaction and barometric depression, according as the spheroid is turned against the direction of the earth's rotation, or the contrary. Hence there result, besides the primary low barometer at syzygies, and high at quadratures, secondary low at 60° and 240°, secondary high at 150° and 330°, and tertiary high and low at some less definitely marked intermediate points. (Op. citat., p. 398.) I designate the tides as primary, secondary, and tertiary, not in the order of relative magnitude, but of stability and simplicity. The

currents produced by these disturbances apparently give rise to magnetic perturbations, which, although far less regular in their character than those of the barometer, furnish some indications of obedience to similar laws. (Ibid., pp. 434–8).

## IV.

If any direct magnetic disturbance is occasioned by differences of gravitation towards the sun, it is reasonable to suppose that the instantaneous transmission of the gravitating force excites a magnetic current at the same instant in the æther. If M is the sun's mass and R its distance, its attraction is $M \div R^2 = 354936 \div 23000^2 = .00067$. Since this attractive force is added to that of the earth at midnight and subtracted at noon, the solar attraction-disturbance produces a daily difference in the tendency of a particle of air towards the earth's centre equivalent to .00134 of the total force. The solar-diurnal magnetic disturbance between the same hours, as shown by Table II, amounts to .00138 of the total force.

## V.

Each particle of air may be considered as a planet revolving about the sun in an orbit that is modified by elasticity, terrestrial attraction, &c. In consequence of these disturbances, there is alternately a fall of twelve hours towards the sun between midnight and noon, and a rise from noon to midnight. From the nature of accelerating forces, the mean attraction-intensity disturbance should be found at $12^{h.} \div \sqrt{2} = 8^{h.}\ 29'$ from midnight, and, if my hypothesis of a connection between gravitation and magnetism is correct, the mean daily disturbance of magnetic intensity should take place at the same time. Such is very nearly the case, the morning mean occurring about $8^{h.}\ 44'$ after midnight, and the afternoon mean $8^{h.}\ 18'$ before midnight. The average daily mean is therefore $8^{h.}\ 31'$ from midnight, differing but $2'$ from the theoretical mean. The apparent retardation of $13'$, in consequence of inertia and rotation, will be referred to in discussing Props. VIII and IX.

During the intervals between the extremes and means, the æthereal currents and eddies are so variously affected as to render the task of precisely calculating either the particular values of the several disturbances, or their aggregate, extremely difficult, if not altogether impossible. We may, however, by a reference to the following table, easily discover some additional evidences of the dependent relation of magnetism to gravity. The theoretical variations are computed on the hypothesis that the magnetic differences vary as the square of the time, counting from midnight, the commencement of the fall. The barometric differences from the mean are given in ten-thousandths of an inch, the hourly thermometric differences in degrees of Fahrenheit.

TABLE IV. SOLAR-DIURNAL MAGNETIC, BAROMETRIC, AND THERMOMETRIC VARIATIONS AT ST. HELENA

| Astronomical Time. | (A). Observed Magnetic Variations. | (C). Hourly Differences. | (B). Theoretical Magnetic Variations. | (D) Hourly Differences. | A—B. | C—D. | Observed Barometric Differences. | Hourly Thermometric Differences. |
|---|---|---|---|---|---|---|---|---|
| $0^h$. | +.00095 | —.00013 | +.00093 | —.00021 | +.00002 | +.00008 | +.0164 | +.339 |
| 1 | +.00082 | —.00022 | +.00072 | —.00020 | +.00010 | —.00002 | —.0002 | +.215 |
| 2 | +.00060 | —.00020 | +.00052 | —.00018 | +.00008 | —.00002 | — 0161 | —.307 |
| 3 | +.00040 | — 00020 | +.00034 | —.00015 | +.00006 | —.00005 | —.0268 | —.531 |
| 4 | +.00020 | —.00019 | +.00019 | —.00014 | +.00001 | —.00005 | —.0300 | —.934 |
| 5 | +.00001 | —.00016 | +.00005 | —.00013 | —.00004 | —.00003 | —.0259 | —1.054 |
| 6 | —.00015 | —.00011 | —.00008 | —.00010 | —.00007 | —.00001 | —.0179 | —.717 |
| 7 | —.00026 | —.00010 | —.00018 | —.00008 | —.00008 | —.00002 | —.0057 | —.336 |
| 8 | —.00036 | —.00005 | —.00026 | —.00007 | —.00010 | +.00002 | +.0078 | —.239 |
| 9 | —.00041 | —.00004 | —.00033 | —.00004 | —.00008 | +.00000 | +.0182 | —.162 |
| 10 | —.00045 | .00000 | —.00037 | —.00003 | —.00008 | +.00003 | +.0240 | —.175 |
| 11 | —.00045 | +.00002 | —.00040 | —.00001 | —.00005 | +.00003 | +.0204 | —.161 |
| 12 | —.00043 | +.00002 | —.00041 | +.00001 | —.00002 | +.00001 | +.0092 | —.146 |
| 13 | —.00041 | +.00003 | —.00040 | +.00003 | —.00001 | +.00000 | —·0044 | —.182 |
| 14 | —.00038 | +.00002 | —.00037 | +.00004 | —.00001 | —.00002 | —.0175 | —.134 |
| 15 | —.00036 | +.00001 | —.00033 | +.00007 | —.00003 | —.00006 | —.0259 | —.089 |
| 16 | —.00035 | +.00001 | —.00026 | +.00008 | —.00009 | —.00007 | —.0271 | —.088 |
| 17 | —.00034 | +.00002 | —.00018 | +.00010 | —.00016 | —.00008 | —.0210 | —.027 |
| 18 | —.00032 | +.00012 | —.00008 | +.00013 | —.00024 | —.00001 | —.0084 | +.204 |
| 19 | —.00020 | +.00023 | +.00005 | +.00014 | —.00025 | +.00009 | +.0077 | +.707 |
| 20 | +.00003 | +.00031 | +.00019 | +.00015 | —.00016 | +.00016 | +.0227 | +1.117 |
| 21 | +.00034 | +.00029 | +.00034 | +.00018 | —.00000 | +.00011 | +.0342 | +.943 |
| 22 | +.00063 | +.00022 | +.00052 | +.00020 | +.00011 | +.00002 | +.0363 | +.943 |
| 23 | +.00085 | +.00010 | +.00072 | +.00021 | +.00013 | —.00011 | +.0296 | +.814 |

It therefore appears that

1. The night fluctuations of magnetic force (except during the four hours after sunset) are, as their supposed relations to gravity would lead us to expect, comparatively insignificant.

2. The magnetic influence of the atmospheric rotation-tide is perhaps shown by the convergence of the lines of equal magnetic disturbance near the hours of normal high barometer, and their divergence near the hours of low barometer. Thus, in the night, the difference of force is as great in the two hours between 9 and 11 P.M., as in the three hours between 2 and 5 A.M.; and in the day the difference in the three hours between 8 and 11 A.M. is two greater than in the four hours between 1 and 5 P.M.

3. The abnormal hourly differences (C—D) appear to be intimately connected with the barometric tides, having, like the latter, and at corresponding hours, two quarter-daily phases of excess and two of deficiency.

4. Not only do the times of mean baric and magnetic disturbance correspond very closely, as we have seen, but the observed hourly magnetic variation at the times of solar low-tide (6–7 A.M. and 6–7 P.M., allowing for the disturbance of rotation) is nearly the same as the theoretical variation (C—D = —1 in each instance).

5. In the hour immediately following the times of solar mean-tide ($3^{h.}$, $9^{h.}$, $15^{h.}$, and $21^{h.}$), the average theoretical and observed hourly differences are the same.

| | 3 P.M. | 9 P.M. | 3 A.M. | 9 A.M. | Average. |
|---|---|---|---|---|---|
| C, . . . . . . | —20 | —4 | +1 | +29 | 1.5 |
| D, . . . . . . | —15 | —4 | +7 | +18 | 1.5 |
| C—D, . . . . . | —5 | 0 | —6 | +11 | 0 |

6. If any further evidence were necessary to convince us that the barometric and magnetic disturbances are more directly dependent upon rotation, and the consequent continually changing position of each point of the earth's surface relatively to the sun, than upon mere changes of temperature,* it might be drawn from a comparison of columns 7, 8, and 9, in Table IV, or obtained by taking the half-daily barometric differences at each hour, as below:

| | 0 h. | 1 h. | 2 h. | 3 h. | 4 h. | 5 h. | 6 h. | 7 h. | 8 h. | 9 h. | 10 h. | 11 h. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P.M., . | 164 | —2 | —161 | —268 | —300 | —259 | —179 | —57 | 78 | 182 | 240 | 204 |
| A.M., . | 92 | —44 | —175 | —259 | —271 | —210 | —84 | 77 | 227 | 342 | 363 | 296 |
| | 72 | 42 | 14 | —9 | —29 | —49 | —95 | —134 | —149 | —160 | —123 | —92 |

* I speak here merely of the *disturbances*. The total force of terrestrial magnetism, according to my theory, is principally dependent upon the gravitation of temperature currents; but I think that the daily perturbations, both of the magnet and barometer, are determined almost entirely by changes of position consequent upon rotation.

## VI.

The prompt disturbing action of the sun upon the magnetic needle was illustrated in the discussion of Prop. V. The slower operation of the moon may be inferred from the table of lunar-daily magnetic tides, attached to my communication of October 21, 1864 (Op. cit., p. 433). The retardation of the barometric action will become evident from the discussion of the three following propositions.

## VII.

Since the solar and lunar currents are mainly determined by differences of gravitating force, either under the form of differences of pressure or of tidal flow, it seems very probable that the ratio of the barometric to the magnetic disturbance of each luminary may be some function of the relative barometrical and tidal effects of the two bodies. This hypothesis is confirmed by the fact that the lunar-diurnal variations, both of the magnet and of the barometer, exhibit two high and two low daily tides, while the solar-diurnal magnetic variation, like the temperature-tide of the barometer, has only one maximum and one minimum in twenty-four hours.

Let $A' =$ the solar-differential tidal force.

$B' =$ the diurnal barometric variation.

$M' =$ the diurnal magnetic variation.

Let $A''$, $B''$, $M''$ represent the corresponding lunar elements.

If the modern physical hypotheses are correct, and the forces that produce A, B, and M, are all forms of motion, it is probable that some simple relationship may exist between them. In endeavoring to ascertain that relationship, we readily discover that

$$A' < A''$$
$$B' < M'$$
$$B' > B''$$
$$B'' > M''.$$

These inequalities, together with the fact that the solar currents are developed in air that is disturbed by the greater attractive energy of the moon, and the lunar currents in air that is disturbed by the more powerful barometric action of the sun, suggest the following hypothetical equivalent proportions:

$$B' : B'' :: \sqrt{A' \, M'} : \sqrt{A'' \, M''}$$
$$A' : A'' :: B'^2 \, M'' : B''^2 \, M'$$
$$M' : M'' :: B'^2 \, A'' : B''^2 \, A'.$$

From the same considerations we may readily infer a probability that

$$\frac{B'}{M'}=\frac{A'}{A''} \qquad (1)$$

$$\frac{B''}{M''}=\frac{B'}{B''} \qquad (2)$$

and that, therefore, $B''$ is a mean proportional between $B'$ and $M''$. From these equations the foregoing proportions may be reproduced by dividing (1) by (2), and multiplying the quotient by the identical equation $\frac{B'}{B''}=\frac{B'}{B''}$

According to Major-General Sabine's tables (St. Helena Obs., v. ii, p. lxi), there is a solar maximum, measured in parts of the total force, of +.00095 at noon, and a solar minimum of —.00045 at 11 P.M.:

$$\therefore \qquad M'=.0014 \qquad (3)$$

The lunar tide is so modified by rotation that its true value can perhaps be best ascertained by adding the tides at equal distances from the lunar meridian (Ibid., p. lxii), and taking their average.

LUNAR-DIURNAL MAGNETIC VARIATION IN MILLIONTHS OF THE TOTAL FORCE.

| | 0 h. | 1 h. | 2 h. | 3 h. | 4 h. | 5 h. | 6 h. | 7 h. | 8 h. | 9 h. | 10 h. | 11 h. | 12 h. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Before Lunar M., | +5 | —1 | +4 | —2 | —5 | —5 | —6 | —3 | —2 | —1 | +14 | +15 | +16 |
| After Lunar M., | +5 | —1 | —5 | —6 | —7 | —6 | +1 | +1 | —2 | +18 | +25 | +22 | +16 |
| MEAN TIDE, | +5 | —1 | —.5 | —4 | —6 | —5.5 | —2.5 | —1 | —2 | +8.5 | +19.5 | +18.5 | +16 |

We thus obtain an average low tide of —.000006 at 4 h., and a high tide of +.0000195 at 10 h., which gives

$$M''=.0000255 \qquad (4)$$

The values of $B'$ and $B''$, as deduced from the tables presented at the meeting of July 15, 1864 (Proc. A. P. S., v. ix, pp. 406, 409), are

$$B'=.016 \text{ in.} \qquad (5)$$

$$B''=.00365 \text{ in.} \qquad (6)$$

Dividing by 28.2821, the mean height of the barometer, in order to obtain results in terms of the total barometric pressure, we have

$$B'=.00056573 \qquad (7)$$

$$B''=.0001291 \qquad (8)$$

The relative values of $A'$ and $A''$ have never been precisely determined. Probably the

latest and most correct estimate is the one given in the New American Cyclopedia, article "Tides," according to which, if we assume

$$KA' = 1 \qquad (9)$$
$$KA'' = 2.55 \qquad (10)$$

Of the homologous quantities contained in (1) (2), it is fairly presumable that those of the greatest magnitude (B′, M′) have been most precisely estimated. Assuming their accuracy, we have

1. If (8) be supposed correct,

$$M'' = .00002944 \qquad (11)$$
$$\frac{A'}{A''} = \frac{1}{2.475} \qquad (12)$$

2. If (4) be supposed correct,

$$B'' = .00012 \qquad (13)$$
$$\frac{A'}{A''} = \frac{1}{2.475} \qquad (14)$$

3. If M′ and B″ are required, (4), (9), and (10) being supposed correct,

$$M' = .00144 \qquad (15)$$

and the value of B″ is the same as in (13).

Other hypotheses might be made, but these are sufficient for illustration. Even the widest discrepancy between theory and observation is much less than might have been reasonably anticipated in measurements of such extreme delicacy, and far within the limits of probable error, as will be seen by the following synopsis:

| | KA′ | KA″ | B′ | B″ | M′ | M″ |
|---|---|---|---|---|---|---|
| Observed, | 1 | 2.55 | .00057 | .00013 | .0014 | .0000255 |
| Theoretical, 1 | 1 | 2.475 | .00057 | .00013 | .0014 | .0000294 |
| Theoretical, 2 | 1 | 2.475 | .00057 | .00012 | .0014 | .0000255 |
| Theoretical, 3 | 1 | 2.55 | .00057 | .00012 | .00144 | .0000255 |

In regard to the first theoretical value of M″, it may be observed that it is very nearly equivalent to the mean between .000032, the extreme excursion of the lunar tide, and .0000255, the mean tide.

It is evident that M′ and M″ are theoretically affected only by the *ratio*, and are independent of the specific magnitudes of A′ and A″. Still the following determination of the values that satisfy the hypothetical formula $B = \sqrt{AM}$, may, perhaps, be interesting:

| | Observ. | Theor. 1. | Theor. 2. | Theor. 3. |
|---|---|---|---|---|
| K | 4374 | 4374 | 4374 | 4499 |
| A′ | .000229 | .000229 | .000229 | .000222 |
| A″ | .000653 | .000566 | .000565 | .000565 |

These several determinations of K approximate to the square of the ratio of the velocity of rotation to the velocity of revolution ($92000000 \div \overline{3963 \times 365.25} = \sqrt{4040}$), and to the ratio of the terrestrial to the atmospheric density, and suggest the propriety of considering the element of density (or of its correlative, the square of the time of molecular diffusion), in connection with both A and M.

The value of $A'$ is nearly a mean proportional between the earth's attraction-intensity and the sun's differential-tidal attraction ($\sqrt{2M \div R^3} = \sqrt{709872 \div 23000^3} = .000242$).

The ratio of $M'$ to $M''$ approximates to the compound ratio of the solar and lunar attraction-intensities and their differential attractions.

## VIII.

The theoretical determination of $M'$ from the joint-consideration of the maxima of the tidal disturbances of equilibrium and the barometric variations (according to the third hypothesis in Prop. VII), is .00144, which is .00004 greater than the extreme observed range of the solar-diurnal magnetic fluctuations. The determination of the same element from the simple consideration of the solar attraction (Prop. IV), was .00134, which is .00004 less than the observed difference of magnetic intensity between the hours of greatest and least attraction. The sum of the two theoretical is precisely equal to the sum of the two observed variations, a result which tends to confirm the opinion that all the disturbances which are attributable to differences of gravitating force, whether initiated by tidal differences, differences of temperature, or rotation, all tend to produce perturbations of magnetic force.

The following comparison appears to show that whatever indirect influence may be exerted upon the magnet by the regular barometric variations, being communicated through the aerial vibrations, is retarded by atmospheric inertia:

| | Max. | Min. | Greatest Half Daily Fluctuations. | Least Half Daily Fluctuations. |
|---|---|---|---|---|
| Barometric pressure, . . . . . | 22 h. | 4 h. | 9—21 h. | 3—15 h. |
| Magnetic force, . . . . . | 0 h. | 10—11 h. | 0—12 h. | 7—19 h. |
| Difference of time, . . . . . | 2 h. | 6 h. | 3 h. | 4 h. |

## IX.

Some evidences of special tidal division have already been given; others may be found in a combination of the morning and evening fluctuations, and an examination of the means, as in

TABLE V. DAILY MAGNETIC TIDES, THEORETICAL AND OBSERVED.

| Hours from Midnight. | Theoretical Gravitation Tide (A). | Theoretical Differential Solar Tide (B). | (A + B)* Theoretical Mean Tide (C). | Observed Mean Tide (D). | Difference (C—D). | Rotation Tide. |
|---|---|---|---|---|---|---|
| 0 | —.00067 | +.00024 | —.00043 | —.00043 | .00000 | |
| 1 | —.00058 | +.00021 | —.00037 | —.00043 | —.00006 | ±.00002† |
| 2 | —.00050¼ | +.00012 | —.00038¼ | —.00041½ | —.00003¼ | ±.00003½ |
| 3 | —.00033½ | .00000 | —.00033½ | —.00038½ | —.00005 | ± 00002½ |
| 4 | —.00016¾ | —.00012 | —.00028¾ | —.00035½ | —.00006¾ | ±.00000½ |
| 5 | —.00009 | —.00021 | —.00030 | —.00030 | .00000 | ∓.00004 |
| 6 | .00000 | —.00024 | —.00024 | —.00023½ | +.00000½ | ∓.00008½ |
| 7 | +.00009 | —.00021 | —.00012 | —.00009½ | +.00002½ | ∓.00010½ |
| 8 | +.00016¾ | —.00012 | +.00004¾ | +.00011½ | +.00006¾ | ∓.00008½ |
| 9 | +.00033½ | .00000 | +.00033½ | +.00037 | +.00003½ | ∓.00003 |
| 10 | +.00050¼ | +.00012 | +.00062¼ | +.00061½ | —.00000¾ | ±.00001½ |
| 11 | +.00058 | +.00021 | +.00079 | +.00083½ | +.00004½ | ±·00001½ |
| 12 | +.00067 | +.00024 | +.00091 | +.00095 | +.00004 | |

Column A contains the hourly differences from mean weight attributable to solar gravitation, with changed signs,—diminution of weight being accompanied with increase of magnetism, and *vice versâ*.

The form of the tide in column B is evidently such as should be determined by solar action. The magnitude of the tide is estimated by comparing the relative amounts of motion down the diagonal and down the arc of a quadrant ($.00067 \times [1 — (\frac{\pi}{4} — \frac{1}{2})] =$ $.00048$). The mean-tidal difference $[(.00067 — .00048) \div 2]$ is very nearly equivalent to the average theoretical inertia-disturbance of weight. The atmospheric inertia at St. Helena (regarding the fluctuations as uniform between two successive hourly observations), produces retardations of 59′, 85′, 26′ and 31′, at $0^h$, $6^h$, $12^h$, and $18^h$, respectively. The mean retardation is 50′, or $\frac{5}{72}$ of a half-day. The daily gravity-variation being .00134, the average variation in $\frac{5}{72}$ of a half-day is .00009 $\frac{11}{36}$, the mean-tidal difference being 00009½.

The signs of the differences (C—D) are apparently determined, as in column A, by the variations of weight; the magnitude, partly by the differences of pressure in the corres-

* Compare Secchi's Third Law, Phil. Mag. [4], 8, 396.

† The upper signs are for the morning, the lower for the evening hours.

ponding day and night hours, and partly by differences of temperature. The sum of the positive is precisely equal to the sum of the negative differences.

The rotation-tide is the residual difference between the observed mean tide and the actual magnetic force. Its relation to the barometric rotation-tide is shown by the signs, which are positive when the diminution of barometric pressure is in the direction of the earth's rotation, and negative when the pressure is increasing in that direction.

The consideration of the moon's disturbance of the atmospheric gravitation is complicated by the magnitude of its differential attraction, the position of the centre of gravity of the terrestrial system, the varying centrifugal force, and other circumstances involved in the lunar theory. Still there are indications, in the following synopsis, of the influence of gravity, sufficiently striking to encourage a hope that our knowledge of the moon's perturbations may be improved by a thorough comparative study of the lunar astronomical, atmospheric, and magnetic tables.

TABLE VI. LUNAR-DAILY DISTURBANCES OF MAGNETIC FORCE AT ST. HELENA, IN MILLIONTHS OF THE TOTAL FORCE.

| Hours, . . . . . . . . | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Before Lunar M., . . | +5 | −1 | +4 | −2 | −5 | −5 | −6 | −3 | −2 | −1 | +14 | +15 | +16 |
| After Lunar M., . . | +5 | −1 | −5 | −6 | −7 | −6 | +1 | +1 | −2 | +18 | +25 | +22 | +16 |
| Mean, . . . . . . . . | +5 | −1 | −0.5 | −4 | −6 | −5.5 | −2.5 | −1 | −2 | +8.5 | +19.5 | +18.5 | +16 |
| Rotation-Tide, . . . | 0 | 0 | ±4.5 | ±2 | ±1 | ±.5 | ∓3.5 | ∓2 | 0 | ∓9.5 | ∓5.5 | ∓3.5 | 0 |

The above table shows that—

1. The moon's attractive force ($M \div R^2 . = .016 \div 60^2 = .00000\dot{4}$), multiplied by the coefficient of its differential attraction (2.55), gives $.000011\dot{3}$, which is nearly the same as the mean meridional magnetic disturbance [$(.000005 + .000016) \div 2 = .0000105$].

2. The increase of magnetism at 12$^{h.}$ is nearly equivalent to the attractive force, multiplied by the square of the distance from the centre of gravity of the system, and divided by the square of the earth's radius ($.00000\dot{4} + 7707^2 \div 3963^2 = .0000168$).

3. There is a tendency to equality of disturbance on each side of the meridian, at 1$^{h.}$ and 8$^{h.}$, as in the solar magnetic tide.

4. The greatest disturbance occurs at the hours of 10$^{h.}$ and 11$^{h.}$ P.M., both in the solar and in the lunar tide.

5. There are some indications of an increase of gravity and decrease of magnetic force when the tidal flow is towards the centre of gravity of the terrestrial system, and *vice versâ.*

6. The rotation-tide has the customary quarter-daily phases of alternate increase and diminution.

## X.

In consequence of the cumulative tendency of partially obstructed waves, as illustrated in the ocean tides, and in the occasional great breakers on a beach, the atmospheric currents are liable to disturbances, which, according to my theory, should produce the phenomena known as "magnetic storms." Since the normal action which we have been considering requires the employment of a constant portion of the magnetic force, the aggregate excess should also be constant, and should therefore present analogous numerical relations.

Mr. Airy, the British Astronomer Royal, has published a paper,* in which he presents the sums and coefficients of magnetic irregularities, deduced from a long series of observations. His aggregates appear to indicate that the disturbing force is a third proportional to two other forces, which may be called, respectively, central and tangential.

Thus in his "Table II, Algebraic Sums of Magnetic Fluctuations (in terms of Horizontal Force) for each Year, from 1841 to 1857, including all Days of Record of Great Magnetical Disturbance," the Mean Disturbance is

| Westerly Force. | Northerly Force. | Nadir Force. |
|---|---|---|
| —.00023 = M. | —.00146 = T. | —.00057 = C. |

Here the proportion T : C : : C : M gives for M a

| | |
|---|---|
| Theoretical value, . . . . . . . . . . . . . . . | —.000222 |
| Observed " . . . . . . . . . . . . . . . | —.000228 |

"Table III. Algebraic Sums of Magnetic Fluctuations (in terms of Horizontal Force) for each Year, from 1841 to 1857, including only those days of Great Magnetic Disturbance, in which Records were made by the three Instruments."

| | |
|---|---|
| Theoretical value of M, . . . . . . . . . . . . . | —.000287 |
| Observed " " . . . . . . . . . . . . . | —.000257 |

"Table VIII. Sums, without regard of sign, of Coefficients of Magnetic Irregularity (in terms of Horizontal Force) for each Year, from 1841 to 1857, including all Days of Record of Great Magnetical Disturbance." The proportion C : T : : T : M gives for M a

| | |
|---|---|
| Theoretical value, . . . . . . . . . . . . . . . | .001218 |
| Observed " . . . . . . . . . . . . . . . | .001203 |

"Table IX. Sums, without regard of sign, of Coefficients of Magnetic Irregularity (in

* "First Analysis of One Hundred and Seventy-seven Magnetic Storms, registered by the Magnetic Instruments in the Royal Observatory, Greenwich, from 1841 to 1857. By GEORGE BIDDELL AIRY, Astronomer Royal."—*Trans. Royal Soc.*, Vol. 153, Art. XXIX.

terms of Horizontal Force) for each Year, from 1841 to 1857, including only those Days of Great Magnetic Disturbance, in which Records were made by the three Instruments."

| | |
|---|---|
| Theoretical value of M, | .001137 |
| Observed " " | .001150 |

The "Westerly Force" I regard as representing the permanent magnetic force produced by the constant flow of currents between each point of the isothermal equator and the poles of greatest cold; the "Northerly" and "Nadir" forces as the tendencies towards the solar meridian, and towards or from the earth's centre, produced by the diurnal solar wave. This assignment appears to be justified by Mr. Airy's observation, that "the Aggregate for the Westerly Force . . . . (taken in comparison with that for the Northerly Force), appears to show that, on the whole, the direction of the Disturbing Force is 10° to the east of south;" p. 628. This indicates a line of mean disturbance about midway between the magnetic meridian (which, at London, is about N. 24° W.), and the solar meridian, or midway between the meridians of decussation in the two sets of principal spirals, to which I have referred.

The law of varying attraction suggests a plausible explanation for the approximate mean proportionality of the barometric to the tidal and magnetic variations. For the ratios of attraction of any planet when in solar conjunction, at quadrature, and in opposition, vary as $(n+1)^2$, $n^2$, and $(n-1)^2$, respectively, the attraction at the mean distance being nearly a mean proportional between the maximum and minimum attractions. The barometric fluctuations are occasioned by variations in the gravitation of the air towards the earth's centre,—the tidal motions, by the influence of distant heavenly bodies,—and the magnetic, according to my hypothesis, by the oscillations of the air and æther in their efforts to restore the unsettled equilibrium. The three disturbances, therefore, must evidently have nearly the same mutual relations as if they were produced by three forces, one centripetal, and the other two centrifugal, the two latter being nearly equal in amount but diametrically opposed in direction.

In our discussion of Prop. V, we found some indications of a lagging of about 13′ in the time of the magnetic mean disturbance, which is about one-fourth of the average barometric retardation. This corresponds very nearly with the ratio of the lunar-barometric to the lunar-magnetic disturbance (4.384), and with Mr. Welsh's determination of the moment of magnetic inertia (4.4696; Phil. Trans., v. 153, p. 297).

The foregoing comparisons have been based almost exclusively on Major-General Sabine's discussions of the St. Helena records. It would be desirable, if it were possible, to confirm them by observations at other stations near the equator; but the need of such confirmation is in great measure obviated, by the variety of ways in which I have shown the

probable connection of gravity and magnetism. At extra-tropical posts, the rotation-tide becomes so preponderating that it is difficult to trace the diminished gravitation- and differential-tides, or to discover any obvious numerical relations between the two great forces. Still, the direction of the gravitating force furnishes so satisfactory an explanation of some of the incidental phenomena of magnetic variation, that I am induced to extend my paper somewhat, in order to direct attention to a field of research in which I believe physicists may reasonably hope to obtain encouraging quantitative results.

Among the pointings to unity of force which have led me to this belief, and have strengthened my conviction that terrestrial magnetism is a simple reaction against disturbances of terrestrial gravitation, are the following: Ampère's discovery of the magnetic effect of electric currents circulating around iron bars; Arago's experiments (which were repeated and extended by Babbage, Herschel, Barlow, Christie, and others), showing that simple rotation produces magnetic disturbances which are governed by fixed laws; the distribution of induced magnetism in masses of iron, which, as I announced at the Society's meeting of April 15, 1864, is the same as would follow from the relative centrifugal motions of different portions of the earth, provided the magnetic axis corresponded with the axis of rotation; Hansteen's suspicion, confirmed by Sabine's practical demonstration* of the influence of the sun upon terrestrial magnetism; Secchi's discovery that "the diurnal excursion of the needle is the sum of two distinct excursions, of which the first depends solely on the horary angle, and the second depends, besides, on the sun's declination,"† and that "all the phenomena hitherto known of the diurnal magnetic variations may be explained by supposing that the sun acts upon the earth as a very powerful magnet at a great distance;"‡ Captain Ross's observations upon the effect of barometric fluctuations on the ocean-level (Proc. Roy. Soc., June 15, 1854);§ Mr. Ferrel's paper on the disturbance of barometric pressure by centrifugal force and friction (Nashville Journal of Medicine and Surgery, 1856, and Mathematical Monthly for 1859, vol. 1, p. 140, sqq.);|| and the various other considerations which I have hitherto adduced in support of my views regarding the connection of rotation with aerial and ætherial currents, and with baric or magnetic perturbations.

The hypothesis that the sun has a specific magnetism, which acts upon the earth by simple induction, like the earlier ones, which attributed terrestrial magnetism to one or

* In his discussions of the observations at Toronto, Hobarton, and St. Helena.

† Phil. Mag. [4] 8, 396. ‡ Ibid. 9, 452. § Ibid. 8, 318.

|| I am glad to learn that Mr. Ferrel has resumed his investigations with special reference to a fuller development of the theory of tidal action, and I have a confident hope that his researches will give us a clearer understanding of the phenomena of magnetic, as well as of oceanic tides.

more powerful magnets, lying nearly in the line of the earth's axis, has been objected to on the ground that it is difficult to understand how any conceivable magnetic intensity, by its simple induction, could produce so great a disturbance as is daily observed.* But if Barlow's theory—that the magnetism is superficial—is correct,† and the reasonable grounds which have led to its general adoption have never been controverted, his idea that it is also in some manner induced, must be regarded as extremely probable. The arguments that have been plausibly urged against an induction analogous to that which takes place between ordinary magnets, have no weight if the inducing force is simple gravitation, and the induced effect is a simple reaction. From the fact that magnetism, like gravity, is a central force, varying inversely as the square of the distance, Secchi's conclusions derive a new interest, and lend encouragement to those who are endeavoring to find additional evidences of kinetic unity.

The daily magnetic variations corresponding, as has been shown, both in character and amount, with the variations of gravitating force, the question may naturally arise, whether a like correspondence can be found in the annual and secular changes. The unknown meteorological agency which is gradually shifting the isothermal lines on which I have supposed the terrestrial magnetism primarily to depend, the want of a sufficiently precise determination of the monthly and annual perturbation-values, and the variety of elements (many of which are still undetermined) that should enter into our equations, render it impossible at present to arrive at quantitative results, but the evidence of a qualitative correspondence is most satisfactory.

Among the conclusions which General Sabine has deduced from his discussions are the following:

1. "That the easterly and westerly disturbances have different hours for their principal development,—that these hours differ most widely in different parts of the globe,—and that we may already perceive some indications of a connection existing between the epochs of greatest development and differences of geographical longitude." (St. Helena Obs., Vol. II, p. cxvii.)

2. "The evidence afforded by each of the three observational elements in regard to annual variation is to one and the same effect. January and June are the months of minimum disturbance; September and April the months of maximum disturbance. The aggregate value of the disturbances in the equinoctial months is about three times as great as in the solstitial months. Of the two equinoctial months, the value is somewhat higher in each element in September than in April; and of the two solstitial months, December is higher than June, also, in each of the three elements." (Toronto Obs., Vol. III, p. lxx.)

* See, inter alia, Phil. Mag. [4] 15, 192 and 27, 384.

† Phil. Transactions, 1831.

3. At Toronto, "in both elements, when the relative proportions are taken,—in the Declination of the aggregate values, in the different months, of easterly and of westerly disturbances; and, in the Vertical Force, of disturbances which decrease and disturbances which increase the force, we find that in both cases the proportions vary from a minimum at the southern solstice to a maximum at the northern solstice, the equinoxes being intermediate. . . . . . . . . The relative proportion of the aggregate values of easterly to westerly disturbances of the Declination, and of disturbances which decrease the Vertical Force to those which increase it, varies from one solstice to the other, approximately, as three to one, and in both elements nearly alike." (Ibid. p. lxxi.)

The first of the above inferences is in precise accordance with the theory that the permanent magnetism of the globe is owing to the gravitation of the thermal currents, which are constantly flowing between the thermal equator and the poles of greatest cold.

The second inference presents two points of interest, each of which has been specially noticed by General Sabine. The first is the increase of disturbances as the earth approaches the sun; the second, the relation of the disturbances to the sun's declination, which is precisely such as my hypothesis would indicate. For, at the equinoxes, the action of solar attraction being in the plane of the earth's rotation, the disturbance-spheroid is of course a maximum; while, at the solstices, the planes diverge most widely, and the spheroid is therefore a minimum.

The third inference is likewise confirmatory of my views, since it shows that in the annual, as well as in the daily perturbations, increasing solar altitude, which increases gravitation-disturbance, also increases magnetic disturbance. The equality of the ratios in the second and third inferences is also noteworthy.

The various coincidences which I have thus brought together, present, to my mind, new and convincing evidence of that unity of force which was one of the earliest tenets of speculative philosophy, but which has only recently been brought within the domain of physical science. The doctrine was taught explicitly by Leucippus and Democritus, and, before their day, by the priests of Isis and Osiris; but its practical demonstration was inaugurated by our own countryman, Benjamin Thompson, Count Rumford, and has been successfully prosecuted by Carnot, Seguin, Mayer, Colding, Joule, Grove, and the daily increasing corps of their collaborators. Through their labors a new impetus has been given to the study of natural philosophy, and we may even be pardoned for indulging the hope, that an increased knowledge of the laws and correlations of force, may lead us to a clearer understanding of the nature of Intelligence, to which all force is subject and subordinate.

## NOTE.

The following extracts show that Secchi's hypothesis of solar magnetic induction was only provisional, and that a fuller report than has yet been published of the details of his investigations is greatly to be desired.

"We, too, in the same spirit, and merely for the purpose, if possible, of combining facts, have supposed the sun to act as a great magnet. The explanations hitherto proposed may be reduced either to thermo-electric currents induced by the sun in the different strata of the earth, or to the electricity developed in the meteorological changes of which the sun is the principal cause. A single reflection seems to exclude these from being *principal* causes of the magnetic diurnal period. The characteristic fact, as we have already noticed, is that the magnetic elements have a double period, diurnal and nocturnal. . . . .

"To this proof in support of the solar magnetic theory, may be added another, already noticed by General Sabine, and worked out by us in § 1 of Part II of this memoir, viz., the opposite action of the sun according to its declination, the inversion occurring exactly at the epoch of the equinoxes; and here another difference will be seen between the effects of thermical and meteorological causes and the magnetic effect of the sun. The former do not reach their extremes for a considerable time after the corresponding astronomical phases, while the latter have an almost exact coincidence with them. . . . . .

"We do not pretend, however, that there are not considerable difficulties in the way of this hypothesis; and although it explains very well certain very singular facts,—as, for example, the interval of six hours between the diurnal maxima and minima, a fact the explanation of which has never, as far as I am aware, been even attempted on any other hypothesis, and which yet is so marked in all the magnetic variations in the mean latitudes; also, the singular exception which it suffers at the equator, becoming simple for the horizontal and for the vertical components, and various other points,—yet we must confess that there are some irregularities which our formulæ do not explain. Of this nature is the fact, that at St. Helena, and generally under the equator, the period for the declination of the needle appears to be rather eight hours than twelve, so that it presents sometimes three maxima. . . . . . .

"The fact mentioned above, that the maxima of the perturbations at Hobarton succeed each other with the same retardation as the other magnetic phases, is one which cannot be explained either by the retardation of the effect of temperatures, or by the condensation of vapor. We cannot conceive how these should account for the general retardation of one hour. It is then a purely magnetic fact, the explanation of which depends on that of the physical cause of solar and terrestrial magnetism. The same may be said of the

greater perturbations at the epochs of the equinoxes, which certainly bear no relation to the state of the atmosphere or to the solar heat. . . . . . . .

"A hypothesis, however, can be found which would conciliate the various facts, viz., that atmospheric changes may generate electricity; but that the direction of the current, which of itself would be indeterminate, may be determined by the magnetic action of the sun. But to expand this further into a hypothesis would be at present premature. We will only say that it is not improbable that the earth is subject to the magnetic action of the sun in a manner unknown to us; but now that magnetic phenomena are developing themselves under so many aspects, we may hope that the explanation of these mysterious actions will soon be found." [Phil. Mag., 4th ser., vol. ix, p. 441 to 450.]

I can already discover, in the gravitation of the various currents which I have pointed out, an explanation, satisfactory to my own mind, of all the phenomena here referred to; but numerous special and novel observations are desirable before attempting any further development of my views.